

Implicit in (21) is an intrinsic speed for rock cutting,

$$c = \frac{k\tau_o}{\mu_r g} , \quad (22)$$

which depends on all four properties of the rock but is independent of the jet. The failure criterion (21) can be rewritten in terms of the intrinsic speed as follows:

$$\tau = \tau_o \left(1 + \frac{v}{c} \sin \theta\right) . \quad (23)$$

When (23) is satisfied, the surface layer of grains is always on the verge of being shorn away.

It is now possible to show a posteriori that the thickness δ of the saturated front is indeed small. Since the two forms of (20) must be equal at $|n| = \delta$,

$$\delta = \frac{k(p_s - p_a)}{v \sin \theta} = \mu_r g \frac{p_s - p_a}{\tau - \tau_o} ,$$

where the second equality follows from (21). But (14) must be satisfied simultaneously, so

$$\delta = g \frac{\mu_r}{\mu_w} \frac{\tau}{\tau - \tau_o} .$$

δ is larger than g provided $\mu_r > \mu_w$, but not much larger unless τ is very nearly equal to τ_o , in which case permeability is unimportant anyway. Thus δ is small in a fine-grained rock.

Equations (23) and (14) give rise to a compatibility condition between the fluid and solid mechanics:

$$\mu_w(p_s - p_v) = \tau_o \left(1 + \frac{v}{c} \sin \theta\right) . \quad (24)$$

It is interesting to inquire whether there are circumstances under which (24) cannot be satisfied. Consider the location $\sin \theta = 1$, where the right-hand side of (24) is maximum, and imagine v increasing indefinitely. Equation (8) suggests that p_s can also rise to any level if R becomes small enough. But equation (8) breaks down when $R \sim d$, and p_s cannot rise above the stagnation pressure P_o of the jet. If P_o lies below a critical value P_c given by

$$P_c = \frac{\tau_o}{\mu_w} (1 + v/c) , \quad (25)$$